Checkerboard paradox

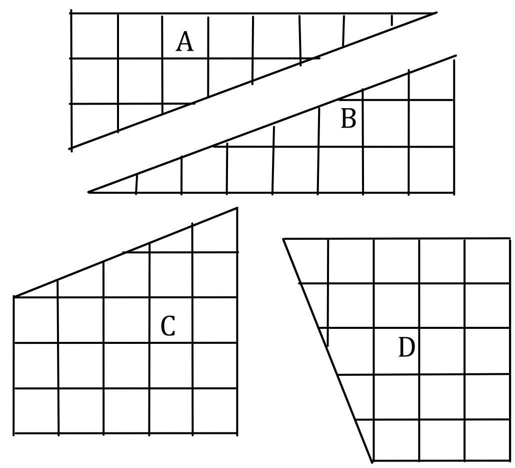
Overview: Once in a while, mathematicians come up against something that really seems impossible on the surface. These seeming "impossibilities" not only cause them to sit up and take notice, but often to create new rules about the way math works, or at the very least understand math a little better.

Be warned, however, that some paradoxes are really false paradoxes, because they do not present ***actual*** contradictions, and are merely "slick logic" tricks. Other paradoxes are real, and these are the ones that shake the entire world of mathematics. There are several paradoxes that remain unsolved today.

Materials

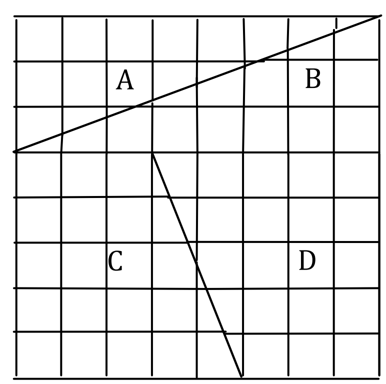
- Pencil
- Template from this page

Activity Cut out the four shapes (A, B, C, and D) so they look like this:

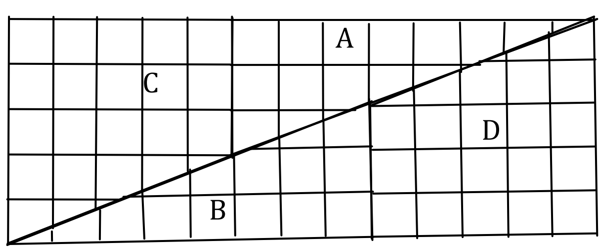


Now can you puzzle the pieces together to make a square? How many little squares are in the big one? ____________

Did you say 64? You're right! Since this is a square of 8 units per side, 8 x 8 = 64 square units.



Now can you make a rectangle like the one below? How many little squares are in the big rectangle?___________

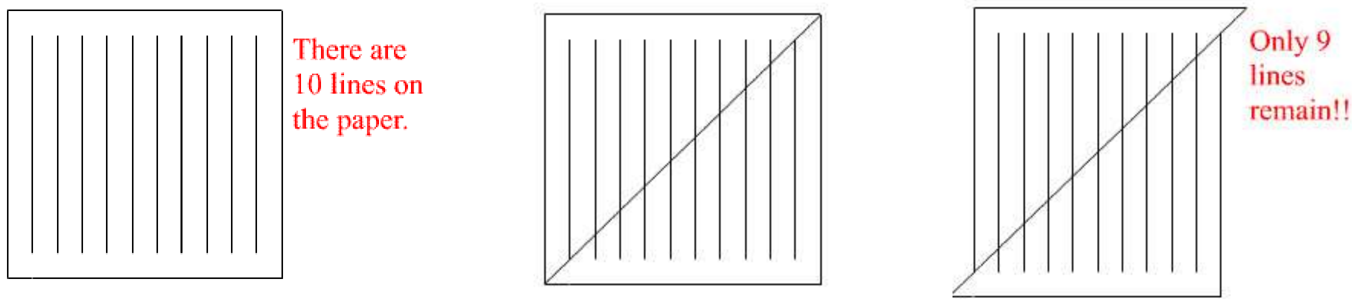


Uh-oh … did you get a different answer? What happened??

The paradox is that when you arrange the pieces in a square, you count up 64 units. When they are in a rectangle, a mysterious 65th box appears.

(Try to figure this out yourself before turning the page!)

Need a hint? I'll give you a hint. Look at this picture:



If you cut the rectangle along the diagonal (the line that magically appears from one corner to the other) and then slide the lower triangle as shown, you can count the number of vertical lines and find that there are only nine! What happened to the tenth?

You can make it magically appear if you slide the lower triangle back to its original position. So… my question to you is: Which is the line that has returned and where does it come from?

The secret is this: there is a progressive decrease in the length of the segments above the diagonal and a corresponding increase in the length of segments below. What happens is that eight of the 10 lines are broken into two segments, then these 16 segments are redistributed to form nine lines, each a trifle longer than before. Because the increase in the length of each line is slight, it is not immediately noticeable. In fact, the total of all these small increases exactly equals the length of one of the original lines. Therefore, there is actually not a line which vanishes.

Now… how would you explain the checkerboard paradox?

Exercises

Your exercise for this lesson is to not only challenge someone else with this problem, but be able to explain it to them in a way that they understand the solution.

Answers to Exercises: Checkerboard Paradox

A close look at the slanted sides of the trapezoidal and triangular pieces shows that they can't be perfectly aligned, and the diagonals of the two smaller rectangles of the 5 x 13 grid are 2 x 5 and 3 x 8, making different slopes for each line. But since the difference is so small (we're talking the difference of 0.025: 2/5 versus 3/8) your eye can be fooled into thinking that the slope of the lines are equal and the grid matches up.