Factorials!

Overview: If I said "3!" would you think the 3 is really excited, or that you have to shout the number?

In fact, it's a mathematical operation called *factorials*, and boy, are they fun! They may seem complicated at first, but they're really a very basic concept. The exclamation point behind a number means that you multiply that number by each successively lower number, in order, until you get to 1. So "3!" would be equal to 3 x 2 x 1 = 6.

Materials

- Pencil
- Paper

Activity: Factorials are useful when you want to know the number of possible combinations or permutations that can be made from a set of objects. A permutation is when the order does matter, and a combination is when it doesn't matter (a permutation is an ordered combination).

Let's do a real-life example: Three friends go into an ice cream shop: Tom, Mary and Jean. Who orders their ice cream first? Second? Last? We can have lots of different ways in which these three people can order their ice cream.

Here are all the possibilities for the three people:

Tom, Mary, Jean

Tom, Jean, Mary

Jean, Mary, Tom

Jean, Tom, Mary

Mary, Jean, Tom

Mary, Tom, Jean

Did you notice how there are six possible arrangements for three people? This can be summarized as 3! = 6. What if there were four people? Then we would have 24 different arrangement styles since 4! =24.

Five friends? Then the answer is 5! = 5 x 4 x 3 x 2 x 1 = 5 x 24 =120.

Do you see how the factorial grows rapidly as you increase the number?

Let's take another example: a deck of 52 cards. The question is, how many different ways can the cards be arranged in the deck? 52! = 52 x 51 x 50 x ….x 3 x 2 x 1 = 80 million trillion trillion trillion trillion trillion

Can you see how factorials start to get really big, really quickly? The card deck is a really great example of this, because with 52 cards the factorial is 52!, which is a HUGE number. There are literally trillions and trillions and *trillions* of ways to arrange those cards. And *that's* why you'll probably never in your lifetime encounter a card deck where the cards are in exactly the same order more than once except when they first come out of the box.

Question: What does 2! = ?

Remember that 2! = 2 x 1 = 2

So then what is 1! =?   It's 1!

But what is 0! ?

Well, I can prove to you that 0! = 1 using the following logic:

$$6 = \frac{6!}{5!} = \frac{6\ x\ 5\ x\ 4\ x\ 3\ x\ 2\ x\ 1}{5\ x\ 4\ x\ 3\ x\ 2\ x\ 1}$$

$$5 = \frac{5!}{4!} = \frac{5\ x\ 4\ x\ 3\ x\ 2\ x\ 1}{4\ x\ 3\ x\ 2\ x\ 1}$$

$$4 = \frac{4!}{3!} = \frac{4\ x\ 3\ x\ 2\ x\ 1}{3\ x\ 2\ x\ 1}$$

$$3 = \frac{3!}{2!} = \frac{3\ x\ 2\ x\ 1}{2\ x\ 1}$$

$$2 = \frac{2!}{1!} = \frac{2\ x\ 1}{1}$$

Using the same pattern we will have:

$$1 = \frac{1!}{0!}$$

But 1 is a whole number, so:

$$1 = \frac{1!}{0!} = \frac{1}{1}$$

For this to be true, then: 0! = 1

Does 0! = 1 make sense to you? If not, that's okay. Just memorize this fact and tuck it away for later. It will come in handy some day in algebra and maybe even for calculus!

Now it's your turn! Work out the exercises below. (You'll find answers at the back of this book.)

Exercises

1. 6!
2. $\frac{6!}{4!}$
3. How many ways can seven different cards arranged uniquely?
4. 0! x 4!
5. 3! x 4!
6. 1! x 5!
7. 2! x 0 x 6!
8. 2! x 4!
9. 3! x 2!
10. 5! x 0! x 1! x 2!

Answers to Exercises: Factorials!

1. 720
2. 30
3. 5040
4. 24
5. 144
6. 120
7. 0
8. 48
9. 12
10. 240