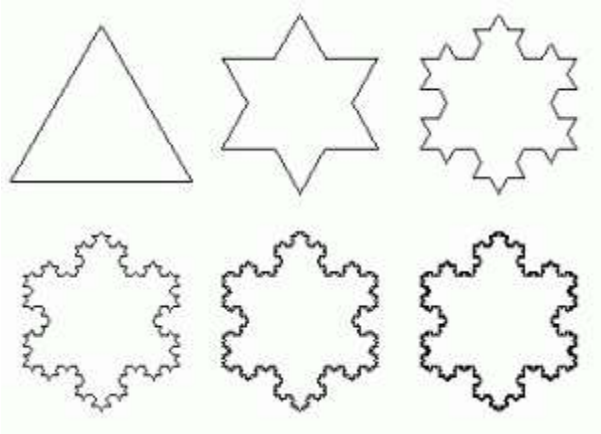
Fractals

Overview: *Fractals* are new on the mathematics scene, however they are in your life every day. Cell phones use fractal antennas, doctors study fractal-based blood flow diagrams to search for cancerous cells, biologists use fractal theory to determine how much carbon dioxide an entire rain forest can absorb.

Fractals are in the mountains, clouds, coastlines, central nervous system, flower petals, sea shells, spider webs… they're everywhere! And the really nifty thing about fractals is that they are not only cool, they're super-useful in our world today.

Many mathematicians today are building on the work pioneered by Karl Weierstrass (1872), Helge von Koch (1904), and Waclaw Sierpinski (1915) to figure ways of using the ideas behind fractals. One of the most interesting parts about fractals is that many ideas about fractals were first thought up of in our lifetime. Many different fields, including medicine, business, geology, clothing fashion, art, and music use ideas about fractals.

Fractals are beautiful (there is something hauntingly stunning about the computer-generated images of objects such as the Mandelbrot set, Julia sets, the Koch snowflake). But that's not all – they are useful in our technology world. However, you'll find that many research mathematicians still roll their eyes at the mention of the word "fractal," mostly because the discussions you'll find out there concerning fractals are missing the most important element – the mathematical content! This is why you'll often find both students and teachers thinking that fractals are reserved only for art and video games, when that's only one side of a multi-faceted concept.

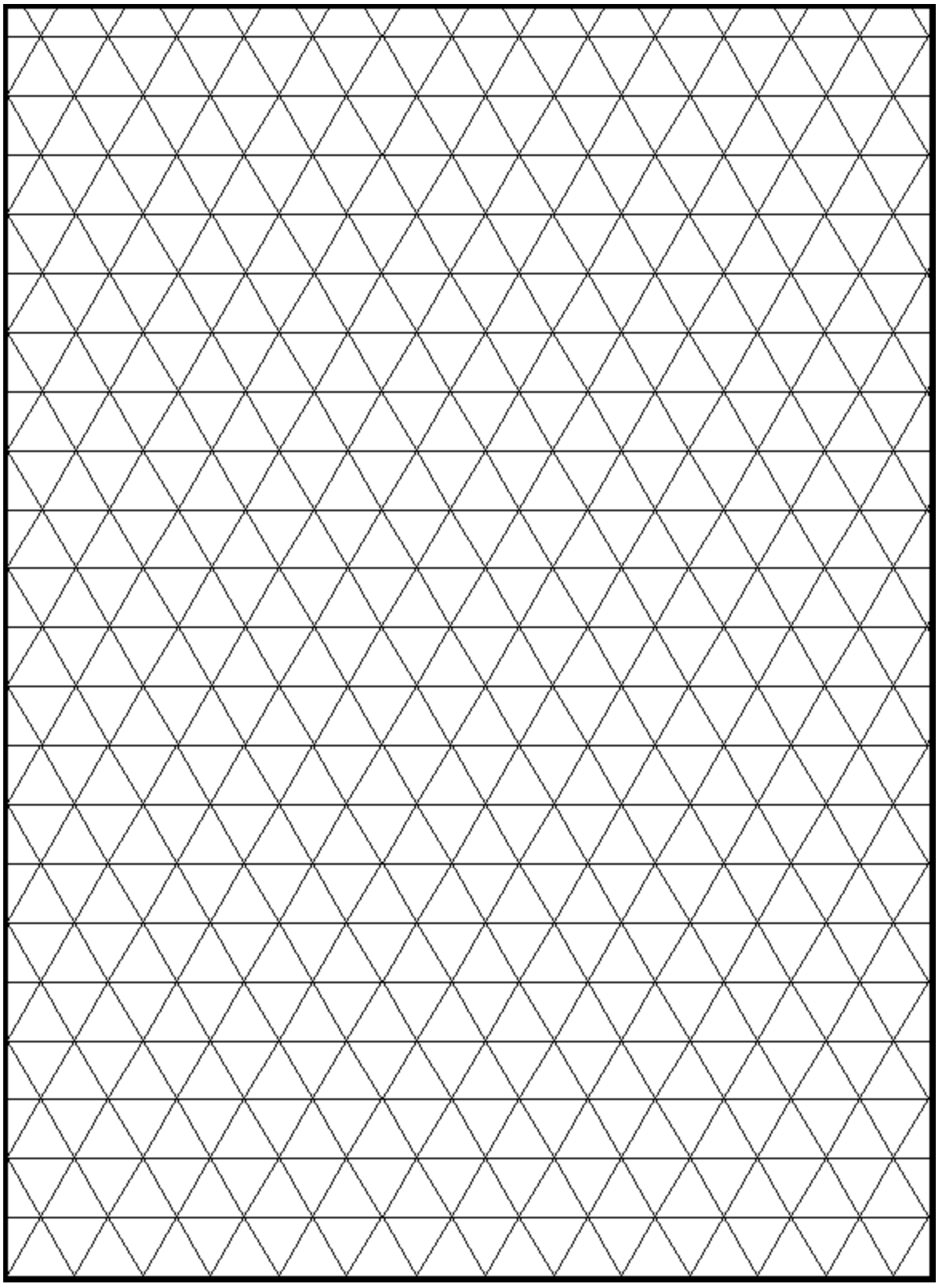


There is solid mathematics behind the pretty pictures – in fact, with a good program, most kids can create their own fractal images after starting with the mathematics (which is often more beautiful than the images themselves!)

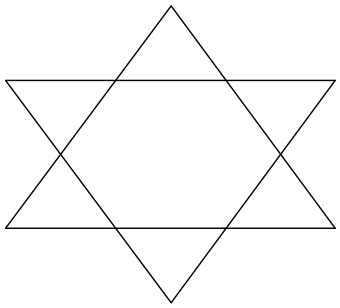
I'm going to help you unravel some of the mystery of fractals while having a lot of fun doing it. There are lots of easy-to-teach topics involving ideas from fractal geometry.

Materials

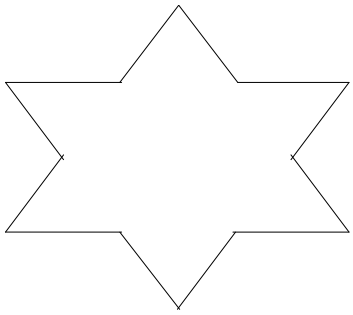
- Pencil
- Paper
- Grid made of triangles



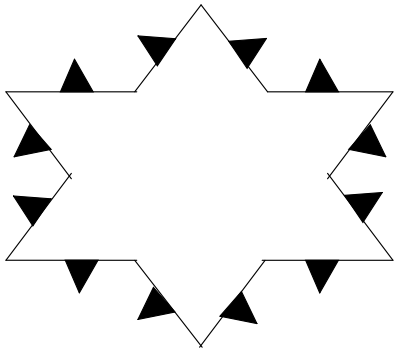
Activity: Look at how I can put together two triangles to make a six-sided star:



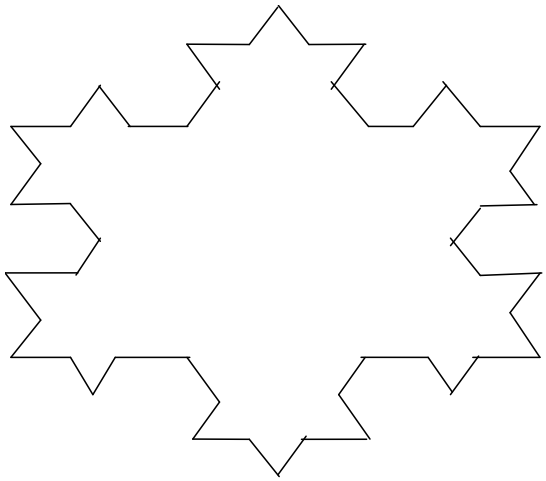
If I erase the inner lines of the triangle, I get the figure shown below:



Now find the middle of each line and add another triangle to each midpoint (dark triangles).



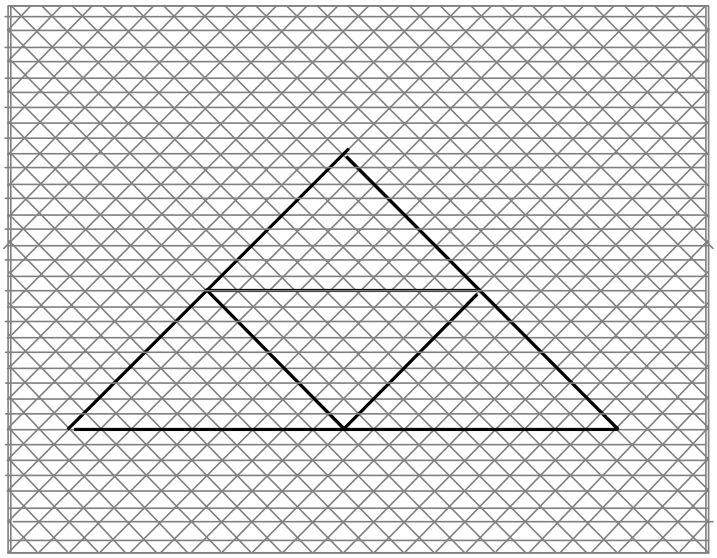
Now erase the lines to get:



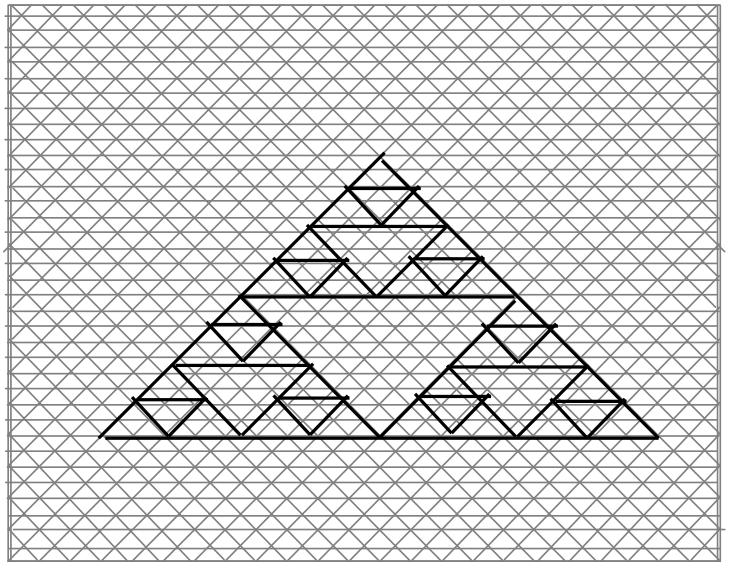
When we continue with adding more and more triangles at the midpoints of every line, you'll discover a complex and detailed structure similar to the one shown here. These are fractals!

The more triangles you add, the more the lines assume a more curved line.

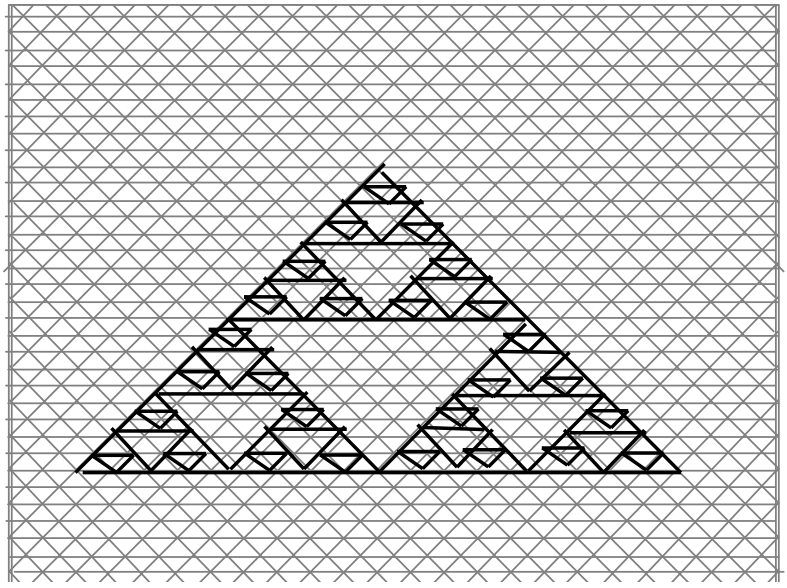
Let's try another way to create a fractal. Use the triangle paper to draw an equilateral triangle. Then divide it into three sections as shown:



Ignore the middle triangle, and now divide the other three smaller triangles as shown:



Again, ignore the middle triangles then divide the other smaller triangles as shown:

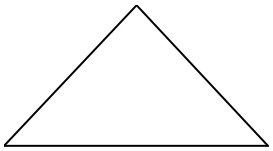


Continue doing this until you make the triangles as small as you can. You've just made a fractal! Notice that in every step, we ignored the middle triangle(s) and then divide the remaining triangles. There's a mathematical formula that can figure out the total area of the un-shaded (ignored) sections of a triangle.

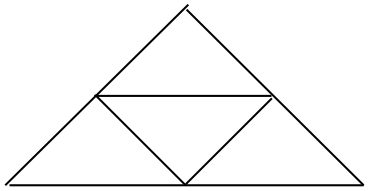
This particular triangle is called *Sierpinsiki* triangle, and it's widely used in structural engineering to model strong structural objects.

Exercises

1. What are the best structures that can describe the structure of human hand, flowers, and peaks of mountains among others?
2. What are the building units of fractals?
3. Identify the name of the following figure.



4. From the above lessons, what is the best type of three-sided figure that can model fractals. How many triangles are you able to identify in the following figures?
5.



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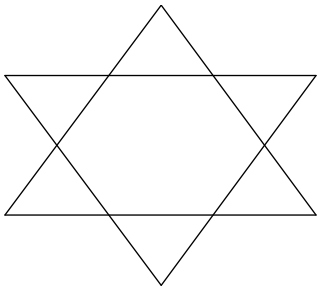
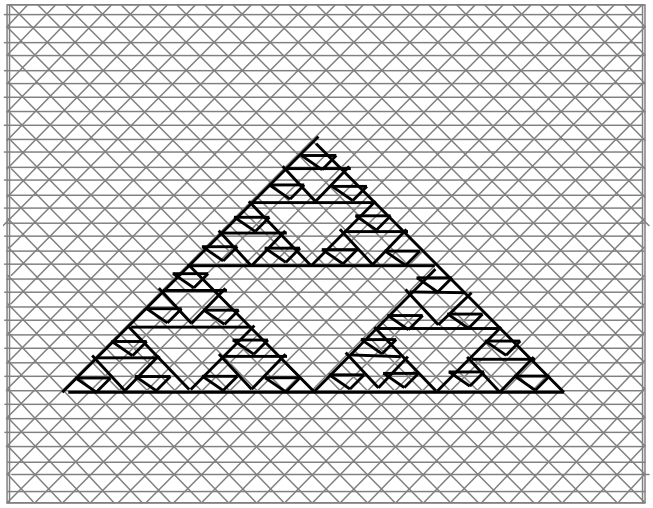


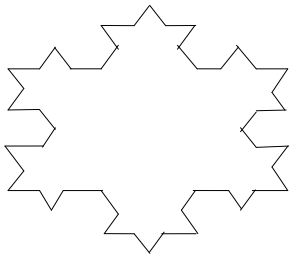
7. What is the name given to the following structure



8. Where is the structure commonly used?

What are the basic procedures that are used in making the following fractals in the following two examples?

9.



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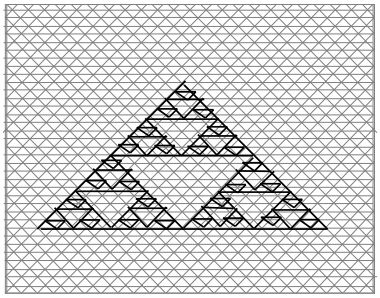

Answers to Exercises: Fractals

1. Fractals
2. A series of known geometrical figures such as triangles, squares and circles
3. Triangles
4. Equilateral triangles
5. 5
6. 8
7. Sierpinsiki triangle
8. Engineering field
9. Erase the middle third of the sides, draw the triangle at the very place and erase the inner lines
10. Divide the triangle into 3, ignore the middle part and repeat the step for the other small remaining parts