Möbius strip

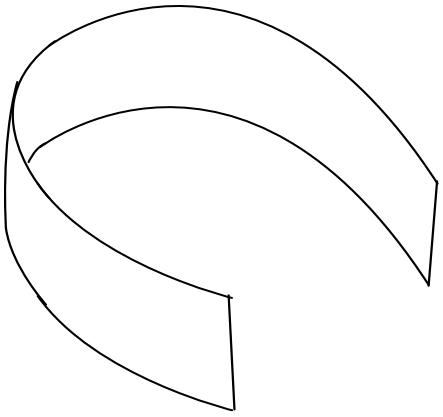
Overview: Although the Möbius strip is named for German mathematician August Möbius, it was co-discovered independently by Johann Benedict Listing, a completely different German mathematician, but at around the same time in 1858. Weird, right? But that's not the only strange thing about the Möbius strip. It's a non-orientable surface. This means it has a path that will take a traveler back to their point of origin. Are you completely confused now?

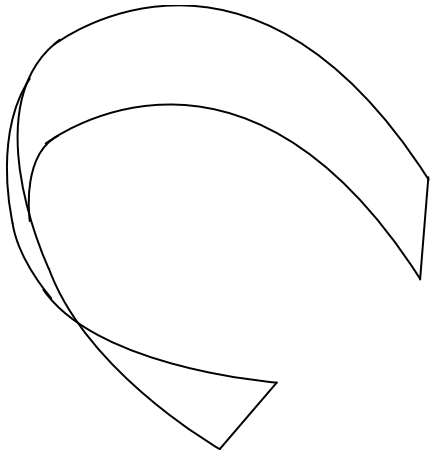
Materials

- Pen
- A pair of scissors
- Paper
- Tape

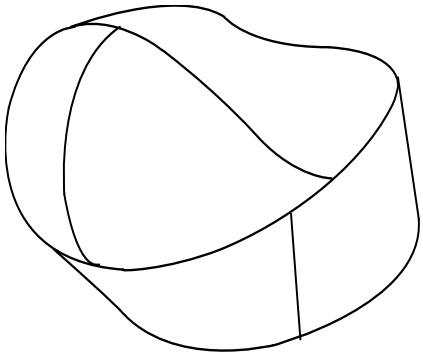
Activity: Let's make one of these mysterious Möbius trips and then we'll play with it!

First, cut out a strip of paper. Before taping it together to make a bracelet, twist one end a half turn like this:





Now connect the ends to form a Möbius strip.

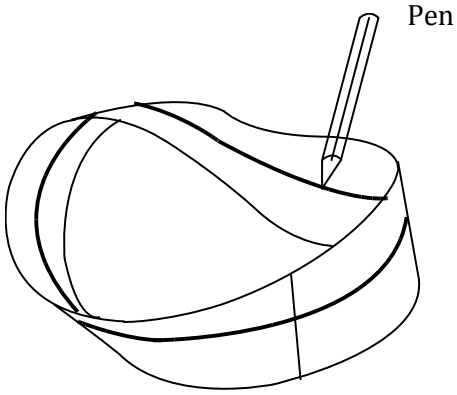


The strip has one side (well, two if you also count the thickness of the paper, but for now you can ignore this since it's so much smaller than the width),and unlike a circular strip which has two sides: the inside face and the outside face.

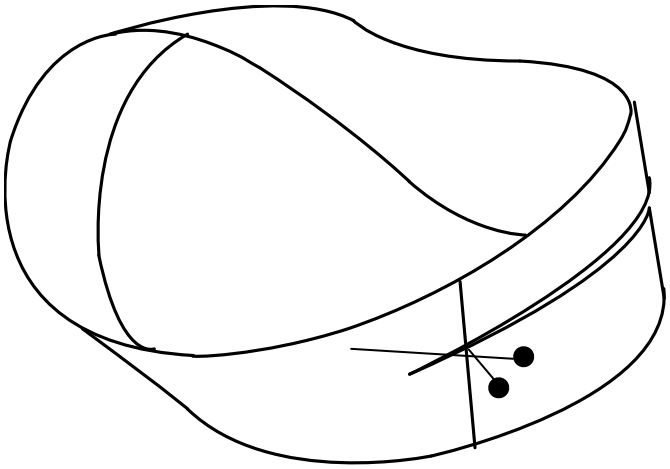
This is where the paradox is. A *paradox* is something that seems to contradict itself. It sort of makes sense, but then again it doesn't *really* make sense. It seems odd that a paradox fits into a math lesson, yet they do exist in mathematics. The Möbius strip is a fun example of a paradox that you can actually touch.

Here's the paradox: take a pencil and start drawing a line along the Möbius strip, and you'll notice that the line gets drawn on both the outside and the inside face, but you haven't lifted your pencil! This line turns out to be just *one* line.

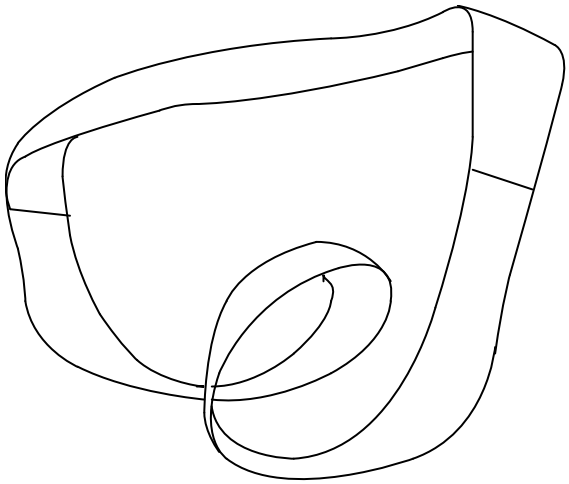
This shows that an object moving along the surface of the Möbius strip will trace one path that appears on its outside and inside face. I once made a slot car race track using flexible track to show kids how the car continuously runs on the track in a circle, but sometimes it's on the inside surface and sometimes it's on the outside!



What happens if we cut the strip in half longways? Grab your scissors and cut it just like it's shown in the diagram:



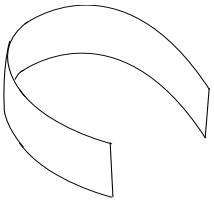
Did you get this shape? A strip with one twist?



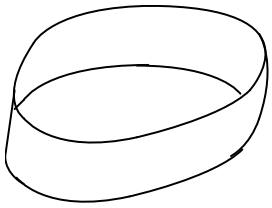
Exercises

Identify the figures shown below:

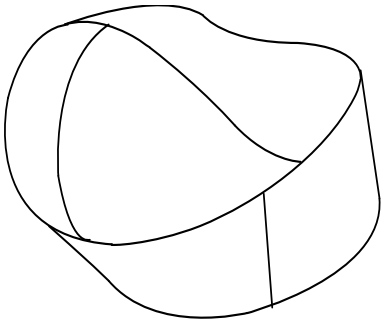
1.



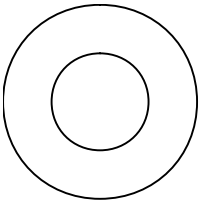
2.



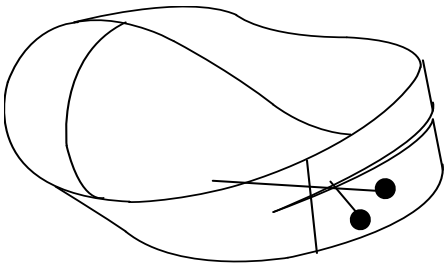
3.



4.



5. How many faces does a Möbius strip have?
6. How many edges does a Möbius strip have?
7. Given a strip of paper, at which angle should one rotate one of its ends before connecting to form a Möbius strip?
8. When a person cuts along the length of the Möbius strip to come up with a longer one, how many twists does he observe in the final strip?
9. Say you want to get two Möbius strips from an existing one. How would you do it?



10. If you cut the Möbius strip along its length as shown in 9 above, how many connected strips would you get?

Answers to Exercises: Mobius strip

1. Paper strip
2. Circular strip
3. Möbius strip
4. Annulus
5. 1
6. 1
7. 180 degree
8. 1
9. None
10. 1