Three doors

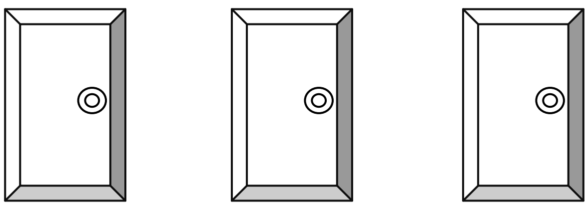
Overview: Ever dream about winning big on a game show? Would it surprise you to learn that there's math behind it all? Probably not, since you've made it this far through this book. Here's the deal: This lesson in probability teaches how to increase our chances of winning in a game.

Materials

Pencil

Paper

Activity: Imagine that you are on a game show with a chance to win a car. There are three doors and the car is behind one of them. You just have to choose the correct door! You can use probability to get a possible advantage in choosing the correct door.



You pick one of the doors at random. Remember that you have equal chances of opening a door of your choice. Since you only have one chance to open one of the three doors, the probability that you open the correct door is:

$$\frac{Number\ of\ chances}{Total\ available\ chances} = \frac{1}{3}$$

The game show host, who knows where the car is, opens a different door (not the one you chose) and shows you an empty room. Should you change your guess?

Mathematically speaking, would you increase your odds of winning if you switched to a different door?

Use the space here to figure out what you should do before moving on to the next page where the answer is.
(No peeking yet!)

Need a hint? Think of it this way: What if there were 100 doors? You pick one. You have a 1/100 chance of guessing correctly with your initial guess. The remaining doors combined together make up a 99/100 chance. The host, who knows where the car is and isn't going to show it to you, opens up 98 of the doors, none of which have the car behind it. The door you initially guessed is still a 1/100 chance, while the last unopened door is 99/100. You should switch your choice!

Not convinced? Imagine you chose door #1, which means you have a 1/3 chance of winning, and a 2/3 chance that the car is hidden behind one of the other doors. But the host gives you a clue by opening a losing door. Notice that if the car is behind door #2, the host will open #3, and if the car is behind #3, door #2 is opened. This is important because when you switch, you win if the car is behind #2 *or* #3, so you win either way! But if you don't switch your choice, you can only win if the car is behind door #1.

Still not convinced? The winning odds of 1/3 on the first choice don't increase to 1/2 just because the host opens up a losing door for you to see. The benefits of switching are really seen if you do this with three cups and a penny. Place a penny under one of the cups (make sure you don't use transparent water glasses!) and play six games that have all the possibilities explored.

You'll notice for the first three games, you choose cup #1 and then switch each time. For the second set of three games, you choose cup #1 and stay (don't switch your choice) each time. The host always reveals an empty cup to you. Here's what you'll find:

	Cup #1	Cup #2	Cup #3	Result
Game 1	Penny	None	None	Switch and you lose.
Game 2	None	Penny	None	Switch and you win!
Game 3	None	None	Penny	Switch and you win!
Game 4	Penny	None	None	Switch and you win!
Game 5	None	Penny	None	Switch and you lose.
Game 6	None	None	Penny	Switch and you lose.

Did you notice that you switch, you win 2/3 of the time, and only lose 1/3 of the time? And when you don't switch, you win 1/3 of the time and lose 2/3?

This is a classic problem that people get confused about. Thinking in terms of 100 doors makes it easier to see how the math works!

This problem and solution was originally published in PARADE magazine in 1990 and 1991.